Closed-Loop Process Identification for Air Separation Unit

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Closed-loop identification is more difficult compared to open-loop identification because of the correlation between the noise and the process inputs. This paper presents a new closed-loop identification method to solve this problem. Based on one closed-loop test, the complete models, including process and disturbance dynamics models, can be identified simultaneously. The finite impulse responses (FIR) of the closed loop systems are first estimated by a recursive algorithm derived from subspace identification method. Then, the fast Fourier transform (FFT) and inverse FFT (IFFT) techniques are used to construct the frequency responses and FIR models, respectively, of process and disturbance dynamics. The proposed algorithm does not require any prior assumptions about the structure of the model. Finally, the case study results for air separation units have demonstrated the effectiveness of the proposed identification method.

1. INTRODUCTION

For the purpose of supplying the needs of high purity oxygen, China Steel runs some stand-alone air separation units (ASUs). The process input is air, coming from the atmosphere, and the products are oxygen, nitrogen and argon. Figure 1 shows a simplified flow diagram for an air separation plant, in which the process can be roughly split into two steps. Before entering the distillation towers (cold box), air purification has to be carried out in order to remove unwanted ingredients, such as dust, moisture, CO₂, and hydrocarbons, from the three products, oxygen, nitrogen and argon. In addition, as the product demand fluctuates significantly in most ASU processes, the operating conditions need to be switched frequently. This switching leads to long transients that disrupt the system performance and profitability². As expected, there is significant economic interest in reducing the operating costs of ASUs through advanced process control technology. So far, the dominating control practice in ASU processes has been to adapt the traditional regulatory controllers to maintain good performance.

For this purpose, the closed-loop identification plays an important role in improving existing underperforming controllers, also enabling the identification of systems that cannot operate in open-loop fashion. The methods, based on the identification of a closed-loop system from which a process model can be obtained using knowledge of the operating controller, have been termed as indirect closed-loop identification. The work presented in this paper is based on this approach.

![Fig. 1. Simplified flow diagram of air separation unit.](image-url)

In this paper, a new closed-loop identification method is proposed. This method can be classified into the joint input-output approach. The complete models,
including process and disturbance dynamics models, can be identified using only one closed-loop test. The algorithm does not require any prior parameterization about the models. Based on testing, the finite impulse response (FIR) models of several closed-loop systems are first estimated by a recursive algorithm derived from subspace identification methods (SIM). The use of SIM for indirect closed-loop identification can bypass the special treatment required for traditional direct closed-loop identification. Also, the recursive algorithm makes the estimation of FIR models more efficient. Then, the fast Fourier Transform (FFT) technique is used to construct the frequency responses of the process and disturbance dynamics. Finally, the FIR models of the process and disturbance dynamics are calculated by the inverse FFT (IFFT) technique.

2. IDENTIFICATION OF FIR MODEL

There are a number of obvious advantages of the FIR model \((h_i, i=1,2,3,\ldots)\) from the viewpoint of system identification \((6)\). For example, the determination of FIR models requires less a priori knowledge than do the parametric models, and this model can be identified more satisfactorily in the presence of noise. For the open-loop stable system, FIR will decay to zero after some \(i>p\). The typical method for identifying the FIR model is the least-squares estimation. However, before preceding with the estimation, the value of the settling time parameter \(p\) must be chosen in advance. To obtain an accurate result, \(p\) should be picked according to the condition \(h_i \approx 0\) for \(i>p\), which may result in some complexities of computation, because, usually, we may need to repeat the solution with progressively increasing \(p\) values until a satisfactory fit has been achieved. Moreover, a large \(p\) increases the computational difficulties associated with high order matrix inversion in the least-squares estimation. To overcome this difficulty, a new FIR estimation method based on part of the subspace identification algorithm is proposed.

2.1 Alternative Method for FIR Estimation

The innovation form of state-space representation of systems is given by

\[
x_{k+1} = Ax_k + Bu_k + Ke_k \\
y_k = Cx_k + Du_k + e_k
\]

where \(x_k \in \mathbb{R}^n, u_k \in \mathbb{R}^n, y_k \in \mathbb{R}^n, \) and \(e_k \in \mathbb{R}^n\) are the system state, input, output, and white noise (innovations), respectively. \(A, B, C, D\) are system matrices with appropriate dimensions, and \(K\) is the steady-state Kalman gain. Based on the innovation form in Eq.(1), an extended state-space model can be formulated as \((1,4)\):

\[
Y_f = \Gamma_N X_f + H^{d}_N U_f + H^{s}_N E_f \quad \text{...............(2)}
\]

where the subscript \(f\) denotes future horizon. The future block Hankel matrices are defined as:

\[
U_f \Delta U_{N(2N-1)+} \Delta \in \mathbb{R}^{n_u N_x j}
\]

\[
H^{d}_N \in \mathbb{R}^{n_y N_x n_u N} \quad \text{...............(3)}
\]

\[
H^{s}_N \in \mathbb{R}^{n_y N_x n_y N} \quad \text{...............(4)}
\]

The extended observability matrix \(\Gamma_N\) is given as:

\[
\Gamma_N = \begin{bmatrix} C & CA \\ CA^{-1} \end{bmatrix} \in \mathbb{R}^{n_y N_x n} \quad \text{...............(5)}
\]

The lower triangular block-Toeplitz matrices \(H_N^{d}\) and \(H_N^{s}\) are given by:

\[
H^{d}_N = \begin{bmatrix} D & 0 & \cdots & 0 \\ CB & D & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ CA^{-2} B & CA^{-3} B & \cdots & D \end{bmatrix} \in \mathbb{R}^{n_y N_x n_u N} \quad \text{...............(6)}
\]

\[
H^{s}_N = \begin{bmatrix} I & 0 & \cdots & 0 \\ CK & I & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ CA^{-2} K & CA^{-3} K & \cdots & I \end{bmatrix} \in \mathbb{R}^{n_y N_x n_y N} \quad \text{...............(7)}
\]

The Kalman state \(X_f\) is unknown, but it can be estimated from past input and output data as \((1)\):

\[
X_f = L_p \begin{bmatrix} Y_p \\ U_p \end{bmatrix} \Delta Y_p W_p \quad \text{...............(8)}
\]

where the subscript \(p\) denotes past horizon, and the past block Hankel matrices are \(U_p \Delta U_{0(N-1)}\) and \(Y_p \Delta Y_{0(N-1)}\). Substituting Eq.(8) into Eq.(2), one can write:

\[
Y_f = L_w W_p + H^{d}_N U_f + H^{s}_N E_f \quad \text{...............(9)}
\]
The subspace matrices \( L_w \) and \( H^d_w \) can be identified from data Hankel matrices using regression techniques such as the least-squares method, provided that the following conditions are satisfied:\(^{(1)}\):

1. The input \( n_t \) is uncorrelated with \( e_k \).
2. The input \( n_t \) is persistently exciting.
3. The number of measurements is sufficiently large (i.e., \( n_t \) is sufficiently large).

With the input-output data, the solution of this least-squares problem is given by:

\[
(L_w \ H^d_w) = Y_f \left( \begin{array}{c} W_p \\ U_f \end{array} \right)^T = Y_f \left( \begin{array}{c} W_p^T \\ U_f^T \end{array} \right) \left( \begin{array}{c} W_p^T \\ U_f^T \end{array} \right)^T
\]\n
\[
= Y_f \left( \begin{array}{c} W_p^T \\ U_f^T \end{array} \right) \left( \begin{array}{c} W_p^T \\ U_f^T \end{array} \right)^T
\]

\[
(10)
\]

The matrix \( L_w \) is useful for the estimation of system matrices \((A, B, C, D, K)\), but it is not needed here because our goal is to identify the FIR which can be extracted from the matrix \( H^d_w \). The relationship between the FIR sequences, \( h_i \), \( i=1,2,3,\ldots \) etc., and the state-space matrices is given as:

\[
h_i = CA^{i-1}B, \quad i = 1, 2, 3, \ldots \]

\[
\]

\[
(11)
\]

Notice that each \( h_i \) is a matrix with dimension \( n_y \times n_w \). From Eq. (6), it is found that the first column of matrix \( H^d_w \) contains the \( N \) sequences of FIR. In other words, the first \( N \) sequences of FIR (i.e. \( h_i, i=0, 1, \ldots, N-1 \)) can be obtained from the solution of \( H^d_w \) in Eq.(10). Notice that the value of \( N \) can be chosen to be small so that the computational difficulty associated with high order matrix inversion is avoided. Furthermore, the whole sequence of FIR can be computed using a recursive algorithm as described in the following section.

### 2.2 Recursive Algorithm for FIR Estimation

After the first \( N \) sequences of FIR have been obtained, the next sequences can be computed in a recursive manner. The extended version of Eq.(9) can be formulated as:

\[
\begin{bmatrix}
Y_{N2N-1}^{2N-1} \\
Y_{N2N-1}^{2N3N-1}
\end{bmatrix} = L_w \begin{bmatrix}
W_p^2 \\
U_w^2
\end{bmatrix}
\]

\[
+ \begin{bmatrix}
H^d_{w2N,11}^{2N,11} \\
H^d_{w2N,12}^{2N,12} \\
H^d_{w2N,21} \\
H^d_{w2N,22}
\end{bmatrix} \begin{bmatrix}
U_{N2N-1}^{2N-1} \\
U_{N2N-1}^{2N3N-1}
\end{bmatrix} + \begin{bmatrix}
H^s_{w2N,11}^{2N,11} \\
H^s_{w2N,21}
\end{bmatrix} E_{N3N-1}
\]

\[
(12)
\]

where \( H^d_{w2N,11} = H^d_{2N,22} = 0 \), \( H^d_{w2N,12} = 0 \), and

\[
H^d_{2N,21} = \begin{bmatrix}
CA^{N-3}B \\
CA^{N-2}B \\
CA^{N-1}B \\
CA^{N-2}B \\
CA^{N-3}B
\end{bmatrix} e^T_{N2N-1}
\]

\[
(13)
\]

The lower block of Eq.(12) can be rewritten as:

\[
Y_{N2N-1}^{2N3N-1} - H^d_{w2N,21} U_{N2N-1} = L_w^2 W_p + H^d_{w2N,21} U_{N2N-1} + H^s_{w2N,21} E_{N3N-1}
\]

\[
(14)
\]

As a result, the matrix \( (L_w^2 W_p + H^d_{w2N,21} U_{N2N-1}) \) can be solved by the right-hand-side of Eq.(10), except that \( Y_f \) is replaced with \( Y_{N2N-1}^{2N3N-1} - H^s_{w2N,21} E_{N3N-1} \). Then, the next \( N \) sequences of FIR (i.e. \( h_i, i=N, N+1, \ldots, 2N-1 \)) are obtained in the first column of \( H^d_{2N,21} \). Likewise, the recursive algorithm, for \( m=3,4, \ldots \), is derived as:

1. The matrix \( (L_w^m H^d_{mN,21}) \) is solved by the right-hand-side of Eq.(10), except that \( Y_f \) is replaced with \( Y_{mN(m+1)N-1} - (H^d_{mN,m2} H^d_{mN,m3} \ldots H^d_{mN,mm}) U_{2N3N-1} \)

\[
(15)
\]

where \( H^d_{mN,mi} \) designates the \((m,i)\) block of matrix \( H^d_{mN} \). Notice that \( H^d_{mN,mi} = H^d_{mN,m1} \) and

\[
H^d_{mN,mm} = H^d_{N}.
\]

2. The FIR sequences \( h_i \), for \( i=(m-1)N, (m-1)N+1, \ldots, mN-1 \), are obtained in the first column of \( H^d_{mN,m1} \).

In this way, the FIR sequences can be computed recursively until all the non-zero FIR parameters are identified. Notice that only one computation of (low order) matrix inversion is needed in this procedure.

### 3. CLOSED-LOOP MODEL IDENTIFICATION

Consider a multi-input, multi-output control system, as illustrated in Fig. 2, where \( u \in \mathbb{R}^{n_u} \) is the process input, \( d \in \mathbb{R}^{n_d} \) is the disturbance input, \( r, y, e \in \mathbb{R}^{n_r} \) are the reference, output, and noise signals, respectively, and \( G_d(s), G_c(s), G(s) \) are the process, disturbance, and controller transfer function matrices, respectively, with appropriate dimensions. The controller \( G_c(s) \) is assumed known and the process transfer function \( G_d(s) \) and disturbance transfer function \( G_d(s) \) are the unknowns to be identified. Under closed-loop control, the algorithm presented in the previous section using \( u \) and \( y \) cannot be directly applied because of the correlation between processes input \( u \) and noise signal \( e \). Therefore, an indirect closed-loop identification method is proposed in this section.
In Figure 2, the process output \( u \) and \( y \) are given by

\[
u = G_c(I + G_p G_c)^{-1}r - G_c(I + G_p G_c)^{-1}G_d \Delta M_{ur}(s)r + M_{ud}(s)d
\]

\[
y = (I + G_p G_c)^{-1}G_p r + (I + G_p G_c)^{-1}G_d \Delta M_{yr}(s)r + M_{yd}(s)d
\]

where \( M_{ur}(s) = G_c(I + G_p G_c)^{-1} \) (or \( M_{yr}(s) = (I + G_p G_c)^{-1}G_p G_c \)) is the closed-loop transfer function relating \( r \) to \( u \) (or \( r \) to \( y \)), and \( M_{ud}(s) = -G_c(I + G_p G_c)^{-1}G_d \) (or \( M_{yd}(s) = (I + G_p G_c)^{-1}G_d \)) is the closed-loop transfer function relating \( d \) to \( u \) (or \( d \) to \( y \)). By testing, the FIR models of \( M_{ur}(s) \) and \( M_{ud}(s) \) (or \( M_{yr}(s) \) and \( M_{yd}(s) \)), i.e. \( h_{ur} \) and \( h_{ud} \) (or \( h_{yr} \) and \( h_{yd} \)), respectively, can be obtained using the algorithm given in Section 2 by taking \([r \ d]^T\) as the system input and \( u \) (or \( y \)) as the system output. Then the frequency responses of \( M_{ur}(s) \) and \( M_{ud}(s) \) (or \( M_{yr}(s) \) and \( M_{yd}(s) \)) can be calculated using FFT on \( h_{ur} \) and \( h_{ud} \) (or \( h_{yr} \) and \( h_{yd} \)), respectively.\(^{(5)}\)

\[
M_{ur}(\omega_k) = \sum_{i=0}^{N-1} h_{ur,i} e^{-j2\pi ik/N}, \quad k = 0, 1, 2, \ldots, \frac{N}{2}
\]

\[
M_{ud}(\omega_k) = \sum_{i=0}^{N-1} h_{ud,i} e^{-j2\pi ik/N}, \quad k = 0, 1, 2, \ldots, \frac{N}{2}
\]

\[
M_{yr}(\omega_k) = \sum_{i=0}^{N-1} h_{yr,i} e^{-j2\pi ik/N}, \quad k = 0, 1, 2, \ldots, \frac{N}{2}
\]

\[
M_{yd}(\omega_k) = \sum_{i=0}^{N-1} h_{yd,i} e^{-j2\pi ik/N}, \quad k = 0, 1, 2, \ldots, \frac{N}{2}
\]

\[
\omega_k = 2\pi k/(NT_s), \quad T_s \text{ is the sampling interval. In addition, higher resolution is obtained in the frequency response by padding the FIR with zeros }^{(5)}.
\]

Denoting the length of FIR after padding as \( N \), the frequency response is obtained at frequency values of

\[
\omega_k = 2\pi k/(NT_s).
\]

However, depending on the type of controller \( G_c \), \( M_{ur}(s) \) may not be strictly proper, which prevents its frequency responses from being correctly calculated using FFT. It is proposed here to use a stable and strictly proper filter \( F(s) \), e.g. a first-order lag. That is, the process input \( u \) is first allowed to pass through \( F(s) \), and the FIR model of the transfer function \( M_{ur}(s)G_c(I + G_c F_c)F \), i.e. \( h_{ur,F} \), is identified. Then the frequency response of \( M_{ur}(s) \) can be correctly calculated using FFT on \( h_{ur,F} \).

As a result, the frequency response of \( G_p(s) \) can be solved as:

\[
G_p(\omega_k) = M_{yr}(\omega_k)(I - M_{ur,F}(\omega_k))^{-1} F(\omega_k)
\]

\[
\omega_k = 2\pi k/(NT_s), \quad k = 0, 1, 2, \ldots, \frac{N}{2} \quad \text{for } \omega_k \text{ in the interval } [-\pi, \pi).
\]

Also, the frequency response \( G_d(s) \) can be solved as:

\[
G_d(\omega_k) = -\left(M_{ur,F}(\omega_k)\right)^{-1} M_{ud}(\omega_k) F(\omega_k)
\]

Consequently, the FIR of the process transfer function, i.e. \( h_{pr} \), can be constructed by calculating the inverse FFT (IFFT) of \( G_p(\omega_k) \), that is:

\[
h_{pr,i} = \frac{1}{N} \sum_{k=0}^{N/2} G_p(\omega_k) e^{j2\pi ik/N}, \quad i = 0, 1, 2, \ldots, N - 1
\]

Likewise, the FIR of the disturbance transfer function, i.e. \( h_{dr} \), can be constructed by calculating the IFFT of \( G_d(\omega_k) \).

With the identified FIR models, the step response models can be directly reconstructed, which are very suitable for designing model predictive controllers. If parametric transfer function models are preferred, many algorithms reported in the literature, e.g. Wang et al.\(^{(3)}\), can be applied to identify parametric models from the step responses.

4. ILLUSTRATIVE EXAMPLE

The proposed identification methods are now applied to typical \( 2 \times 2 \) and ASU processes. The identification accuracy is evaluated by the sum of the squared errors (SSE) between the identified and the actual FIRs.

4.1 Example 1: 2x2 Process

Consider the closed-loop system of Fig.2 with the following process, disturbance, and controller transfer function matrices:
The proposed closed-loop identification algorithm is used to estimate models of process and disturbance dynamics under different values of noise-to-signal ratio (NSR). The system is excited simultaneously from \( r \) and \( d \) with a pseudo-random binary sequence (PRBS) shifting between 1 and -1. The sampling time is taken as 0.1. The FIR models of the closed-loop systems are estimated using the proposed recursive algorithm 10 times, each time for 40 parameters. The filter used is \( 1/(5s+1) \). The calculated SSE of identified results are given in Table 1, and the identified FIRs of process and disturbance dynamics under 20\% NSR are shown in Fig. 3. It can be seen that the identification results are satisfactory in the presence of noise.

### 4.2 ASU Process

For ASU, the pressure drop control in the distillation tower during the product demand fluctuations is one of the most important factors. Figure 4 shows the response of the crude argon column of the ASU. The proposed closed-loop identification algorithm is used to estimate models of process. The filter used is \( 1/(5s+1) \). The impulse response of the identified result is shown in Fig. 5. The identified result is shown in Fig. 6. It can be seen that the identification result demonstrates the effectiveness of the proposed identification method.

<table>
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<tr>
<th>NSR</th>
<th>0%</th>
<th>20%</th>
<th>50%</th>
<th>100%</th>
</tr>
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<tr>
<td>( G_{p11} )</td>
<td>2.51e-7</td>
<td>8.40e-5</td>
<td>4.00e-4</td>
<td>0.0015</td>
</tr>
<tr>
<td>( G_{p12} )</td>
<td>8.88e-7</td>
<td>6.08e-4</td>
<td>0.0034</td>
<td>0.0132</td>
</tr>
<tr>
<td>( G_{p21} )</td>
<td>7.06e-7</td>
<td>8.07e-5</td>
<td>4.89e-4</td>
<td>0.0019</td>
</tr>
<tr>
<td>( G_{p22} )</td>
<td>1.68e-6</td>
<td>7.36e-4</td>
<td>0.0072</td>
<td>0.0174</td>
</tr>
<tr>
<td>( G_{d11} )</td>
<td>1.09e-7</td>
<td>1.22e-5</td>
<td>6.68e-5</td>
<td>2.53e-4</td>
</tr>
<tr>
<td>( G_{d21} )</td>
<td>3.70e-6</td>
<td>2.66e-5</td>
<td>1.55e-4</td>
<td>6.14e-4</td>
</tr>
</tbody>
</table>

**Table 1** The calculated SSE of proposed algorithm under different values of NSR for Example 1

**Fig. 3.** Identified FIRs of process and disturbance dynamics under 20\% NSR for Example 1.
5. CONCLUSIONS

This paper presents a new identification method for closed-loop systems. The models of process and disturbance dynamics can be identified from one closed-loop test. The FIR of the closed-loop systems is first estimated by a recursive algorithm derived from subspace identification method. Then, the FFT and IFFT techniques are used to construct the frequency response and FIR of the process and disturbance dynamics. Simulation results have demonstrated the effectiveness of the proposed identification method. The algorithm can be used in conjunction with any control design method to redesign the feedback control system to improve performance.

REFERENCES